acres, and the perimeter of triangle in linear measure equals the area in square measure. Find the length of the chord, the latus-rectum of the parabola, and the dimensions of the triangle.

Solution by the PROPOSER.

Let $PQ=2y$, $BC=x$, $\angle EBC=\theta=\frac{1}{4}\pi$. $\therefore y^2=4ax/\sin^2\theta=8ax$.

$\therefore \text{Area}=2\sin\theta\int_0^x ydx=4\sqrt{a}\int_0^x x^{\frac{1}{2}}\,dx=(\frac{8}{3}a^{\frac{1}{2}}x^{\frac{3}{2}}=\frac{1}{3}(2xy\sqrt{2})$.

$\therefore \frac{1}{3}(2\sqrt{2}xy)=4800$ square rods, or $xy=3600\sqrt{2}$ ....(1).

If G be the center of gravity, then $BG=3x/5$.

$PD=\frac{1}{2}y\sqrt{2}$, $GD=GC-DC=\frac{2}{5}x-\frac{1}{2}y\sqrt{2}=\frac{1}{10}(4x-5y\sqrt{2})$.

$PG=\sqrt{(PD^2+GD^2)}=\frac{1}{10}\sqrt{(100y^2+16x^2-40xy\sqrt{2})}$.

$\therefore PG=\frac{1}{10}\sqrt{(100y^2+16x^2-2800)}$,

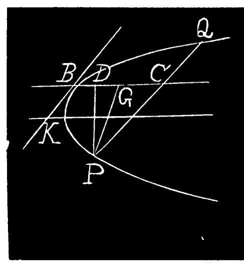
$\frac{1}{2}PD.DG=\frac{1}{40}(4xy\sqrt{2}-10y^2)\frac{1}{4}(2880-y^2)$.

$\therefore \frac{1}{10}\sqrt{(100y^2+16x^2-2800)}+\frac{2}{5}x=\frac{1}{4}(2880-y^2)$.

$\therefore 2\sqrt{(100y^2+16x^2-2800)}=14400-5y^2-8x$.

$\therefore 25y^4-144400y^2+80xy^2-230400x$

$+208512000=0$ ....(2).



(1) in (2) gives

$y^5-5776y^3+11520\sqrt{2}y^2+8340480y=33177600\sqrt{2}$,

$y^5=5776y^3+16291.740672y^2+8340480y=46920213.13536$.

$\therefore y=5.6903$ *nearly*, $2y=11.3806$, length of chord.

$x=894.7101$, $PD=4.0236$, $DG=353.3604$, $PG=353.8833$, area $PDG=711.7673$, $4a=.0181=$latus rectum.

Solved with different results by *C. W. M. BLACK*, and *J. SCHEFFER*.

NOTE on solution of Problem 69, Calculus, April number, page 111: "It seems to me that this solution does not solve this question. The fence prevents the horse from grazing on the ground within it; then, the rope must extend from one end of the major axis around, *outside of the fence*, to the other end, and is twice as long as that half of the fence. Hence the horse may graze around to the end of the minor axis on the other side of the field. The horse, starting from there and keeping the rope tight, will describe a curve as the rope unwinds from the fence, until he arrives at a point opposite the other end of the minor axis, being then half way around: proceeding, he reaches the other end of the minor axis (his starting point) and describes the other half of the curve. *Josiah H. Drummond*.

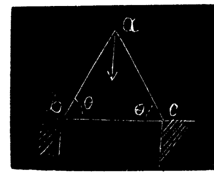
MECHANICS.

64. Proposed by FREDERIC R. HONEY, Ph. B., Instructor in Trinity College, New Haven, Conn.

Let the isosceles triangle abc, whose plane is vertical, and whose base bc is horizontal, and supported at each end b and c, represent three rods jointed at the points a, b, and c. Let any load L be suspended at the vertex a. It is required to find the value of the angle between the sides of the triangle and the base which shall make the sum of the weights of the rods a minimum, the length of the base bc being fixed.

I. Solution by the PROPOSER.

The rods ab and ac are in compression. Let C=number of pounds per square inch the material resists in compression. The rod bc is in tension. Let T=number of pounds per square inch the material resists in tension.



Let W=sum of weight of rods.

Let L=load.

Let w=weight per cubic inch of material employed.

Let θ=angle between the sides of the triangle and the base.

Length of rods $ab+ac=bc\times\sec\theta$.

Tension in rod $bc=\frac{1}{2}L\cot\theta$.

Compression in each of the rods ab and $ac=\frac{1}{2}L\operatorname{cosec}\theta$.

Number of square inches in section area of rod bc needed to resist the tension$=\frac{1}{2}L\cot\theta/T$.

Number of square inches in section area of each of the rods ab and $ac=\frac{1}{2}L\operatorname{cosec}\theta/C$.

Weight of rod bc=length$\times$section area$\times$weight of cubic inch of material $=bc\times(\frac{1}{2}L\cot\theta/T)\times w$.

Similarly, weight of rods $ab+ac=bc\sec\theta\times(\frac{1}{2}L\operatorname{cosec}\theta/C)\times w$.

And $W=bc\times(\frac{1}{2}L\cot\theta/T)\times w+bc\sec\theta\times(\frac{1}{2}L\operatorname{cosec}\theta/C)\times w$

$$=bc\times\tfrac{1}{2}Lw\left(\frac{\sec\theta\operatorname{cosec}\theta}{C}+\frac{\cot\theta}{T}\right).$$

Differentiating, $dW/d\theta=bc\times$

$$\tfrac{1}{2}Lw\left(\frac{\sec\theta\operatorname{cosec}\theta\tan\theta-\sec\theta\operatorname{cosec}\theta\cot\theta}{C}-\frac{\operatorname{cosec}^2\theta}{T}\right).$$

Putting $dW/d\theta=0$, we have

$$bc\times\tfrac{1}{2}Lw\left(\frac{\sec\theta\operatorname{cosec}\theta\tan\theta-\sec\theta\operatorname{cosec}\theta\cot\theta}{C}-\frac{\operatorname{cosec}^2\theta}{T}\right)=0.$$

Dividing by $bc\times\frac{1}{2}Lw\operatorname{cosec}\theta$, and transposing,

$$\frac{\sec\theta\tan\theta-\sec\theta\cot\theta}{C}=\frac{\operatorname{cosec}\theta}{T}.$$

Dividing by $\sec\theta$, $[(\tan\theta-\cot\theta)/C]=\cot\theta/T$.

$T\tan\theta-T\cot\theta=C\cot\theta$.

Dividing by $\cot\theta$, $T\tan^2\theta-T=C$, $\tan^2\theta=[(C+T)/T]$, $\tan\theta=\sqrt{[(C+T)/T]}$.

II. Solution by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering in Agricultural and Mechanical College of Texas, College Station, Texas.

The sum of the weights of the rods will be a minimum when their areas are a minimum, which will occur when the stresses are made a minimum. By resolution of forces we have for the sum of the three stresses in ab, bc and ac, calling the equal angles θ: sum of stresses equals

$$L\left(\frac{1}{\sin\theta}-\tfrac{1}{2}\frac{\cos\theta}{\sin\theta}\right).$$

Equating the first derivative to zero, we get, after reduction, $\cos\theta=\frac{1}{2}$.
Therefore $\theta=60°$ and the triangle is equilateral.

65. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

The distance, parallel to the axis, from the mid-point of a chord to the arc of a parabola, is constant. Show that the center of gravity of all segments formed by the chord is an equal parabola.

I. Solution by the PROPOSER.

Let A be the vertex of the segment ; (h, k) its coördinates ; $5b$ the constant length from A to the center of the chord ; θ the inclination of the chord to the axis ; $y^2=4ax$, the equation to the parabola ; (m, n) the coördinates of the center of gravity of the segment.

Then $h=m=a\cot^2\theta$, $n=k+3b=2a\cot\theta+3b$.

$\therefore$ $\cot\theta=[(n-3b)/2a]=\sqrt{(m/a)}$.

$\therefore$ $m/a=[(n-3b)/2a]^2$. Let $n=p+3b$.

$\therefore$ $p^2=4am$, an equal parabola.

II. Solution by GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

Take the diameter through the mid-point and the tangent at its extremity for axes.

Then $y^2=4cx$, where θ is the angle between the axes and $c=a/\sin^2\theta$.

Since the diameter bisects the area of the segment,

$$x=\frac{\int_o^k 2x\sqrt{(cx)}dx}{\int_o^k 2\sqrt{(cx)}dx}=\tfrac{3}{5}k\ ;\ y=0,$$

where k is the distance from the arc to the mid-point.

But $x=a\cot^2\theta+\frac{3}{5}k$; $y=2a\cot\theta$, referred to the vertex and rectangular axes.

$\therefore$ $(y)^2=4a(x-\frac{3}{5}k)$ or $y^2=4a(x-\frac{3}{5}k)$, which is the equation of an equal parabola with its vertex on the axis at a distance $\frac{3}{5}k$ from the given one.

Also solved by *J. SCHEFFER*.